R gate has input A and 1, therefore it gives output

$$A \oplus 1 = \overline{A} \cdot 1 + A \cdot \overline{1} = \overline{A} + 0 = \overline{A}$$

OR gate has inputs $X = \overline{A}$ and 1, therefore it gives output Z given by

$$Z = X \oplus 1 = \overline{A} \oplus 1 = (A)\,1 + (\overline{A}) \cdot \overline{1}$$
$$= A \cdot 1 + \overline{A} \cdot 0 = \overline{A} \cdot 1 = A.$$

The third and the final XOR has inputs $Z = A$ and A, therefore it gives final output Y, given by

$$Y = A \oplus A = \overline{A} \cdot A + A \cdot \overline{A} = 0 + 0 = 0 \text{ Ans.}$$

(c) $Y = (A + B) \cdot (C + D)$ is implemented using three NOR gates as shown below:

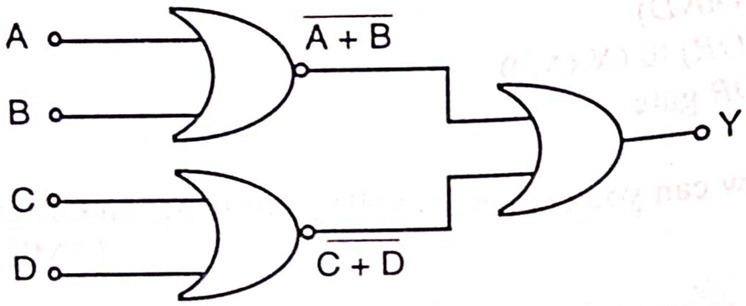

Fig. 2.42

The final output $Y = \overline{\overline{(A + B)} + \overline{(C + D)}}$

$$= \overline{\overline{(A + B)}} \cdot \overline{\overline{(C + D)}} = (A + B)\,(C + D)$$

2.11 KARNAUGH MAP

Boolean switching functions can be simplified and minimized by using postulates of Boolean algebra and De Morgan's theorems. This method of minimization using Boolean algebra has been found to be awkward as it does not follow any specific set of rules, but uses a hit and trial manipulative method as discussed and used in Section 2.1 of this chapter. The map method first proposed and invented by E.W. Veitch and later modified by M. Karnaugh, provides a simple set procedure for minimizing the switching function. This Map method or pictorial representation of the truth table is known as the Veitch-Karnaugh (V-K) map and has become more well-known as Karnaugh map. The Karnaugh map is normally used for representing and minimizing three-variable or four-variable switching function expressions.

As will be shown in succeeding sections, the Karnaugh map is made up of squares. Here each square represents one term. The Karnaugh map is a systematic method for combining the terms and determining minimal expression. Each n variable map consists of 2^n cells or squares. Thus a three variable map will consist of 2^3 or 8 cells or squares and a four-variable map will consist of 2^4 or 16 cells (squares).

C \ AB	00	01	11	10
0	0	2	6	4
1	1	3	7	5

(a) A three-variable Karnaugh map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

(a) A four-variable Karnaugh map

Fig. 2.43

Figure 2.43(a) shows a three - variable Karnaugh map for a switching function $f(A, B, C)$. It has 8 cells or squares. Each square represents a particular decimal value of the function. The first square has AB value of function = 00 and C value = 0, thus it represents the binary number ABC = 000, which is equal to 0 in the decimal system. The extreme last square at the bottom has AB value = 10 and C value = 1, thus it represents a binary number ABC = 101, which is equal to 5 in the decimal system, as is shown in Fig. 2.23(a). Similarly Fig. 2.43(b) shows a four variable Karnaugh map for a switching function $f(A,B,C,D)$. It has 16 cells and squares. Each square, here again represents a particular decimal value of the function, as shown in Fig. 2.43(b).

2.12 CANONICAL FORM 1

Any Boolean expression representing a switching function can be obtained from the truth table by writing the sum of all the terms, which correspond to all those combinations or rows, for which the function attains the value '1'. Each term in the product of the variables. Any variable should be taken in uncomplemented form in the product if it has the value '1' and should be taken in complemented form if it has the value '0'. If in a three-variable function $f(A,B,C)$ the value of A, B and C are 1, 0 and 1 respectively, then the product term will be ABC. A product term which has each of all the variables as factors in either complemented or uncomplemented form is known as minterm.

The switching function expressed as the sum of all the minterms is called the Canonical Sum Of Products (SOP) or disjunctive normal expression.

check to intern ↓ what is SOP?

EXAMPLE 29: Find the Sum Of Products (SOP) form of switching function $f(A,B,C)$, which is represented by the truth table given below:

Table 2.12

Decimal Value	*A*	*B*	*C*	*f*
0	0	0	0	0
1	0	0	1	1
2	0	1	0	1
3	0	1	1	0
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	1

From the above table it is evident that the decimal values for which the function f assumes the value '1' are 1,2,4,7.

Thus the function $f(A,B,C)$ is the sum of these product terms i.e., = $\Sigma(1,2,4,7)$

$$= \Sigma\ (1,2,4,7)$$
$$= 001 + 010 + 100 + 111$$
$$\therefore f(A,B,C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

which is the sum of products (SOP) form of the function.

Maxterm A sum term which has each of all the variables of the switching function in either complemented or uncomplemented form is known as maxterm. The expression expressed as product of all the maxterms for which the function attains the value '0' is known as the canonical product of sums (POS) or conjunctive normal expression. For each maxterm, any variable should be taken in the uncomplemented form if it has value '0' and should be taken in the complemented form if it has the value '1'. For example, if in a three variable function $f(A,B,C)$, values of A,B and C are 0,1 and 0 respectively, then the sum term will be $(A + \bar{B} + C)$.

EXAMPLE 30: Find the Product of Sums (POS) form of the switching functions, which is represented by the truth Table 2.12 of Example 29.

Solution:
From the truth Table 2.12, it is evident that the decimal values for which the function f assumes the value '0' are 0,3,5,6. Thus the functions $f(A,B,C)$ is product of these sum terms, i.e.,

$$f(A,B,C) = \Pi\ (0,3,5,6)$$
$$= (000)\ (011)\ (101)\ (110)$$
$$= (A+B+C)\ (A+\bar{B}+\bar{C})\ (\bar{A}+B+\bar{C})\ (\bar{A}+\bar{B}+C).$$

Thus POS form is the products of those sum combinations for which the function has the value '0'.

When any switching expression is to be expressed in canonical SOP forms, then each term of the expression should be examined and if it is a minterm, then it should be kept as it is. Thus if any term contains all the variables in either complemented or uncomplemented form, then it should be retained. If any particular variable does not occur in any term, then for each variable A, B or C which do not occur, multiply that term by $(A+\bar{A})$, $(B+\bar{B})$ or $(C+\bar{C})$ as the case may be. Then multiply all the terms and eliminate the repeated terms.

EXAMPLE 31: (i) Find the canonical sum of Products (SOP) form for the expression given below:

$$f(A,B,C) = AB + \bar{A}\bar{B} + AC + \bar{A}\bar{C}.$$

(ii) Also find product of sums expression for the above expression.

Solution:

$$
\begin{aligned}
f(A,B,C) &= AB + \bar{A}\bar{B} + AC + \bar{A}\bar{C} \\
&= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + AC(B + \bar{B}) + \bar{A}\bar{C}(B + \bar{B}) \\
&= ABC + AB\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ACB + AC\bar{B} + \bar{A}\bar{C}B + \bar{A}\bar{C}\bar{B} \\
&= ABC + AB\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + ABC + A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}
\end{aligned}
$$

Eleminating ABC and $\bar{A}\bar{B}\bar{C}$ once as they are repeated twice, we get

$$
\begin{aligned}
f(A,B,C) &= ABC + AB\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}C + \bar{A}B\bar{C} \\
&= 111 + 110 + 001 + 000 + 101 + 010 \\
&= \Sigma(7,6,1,0,5,2) \\
&= \Sigma(0,1,2,5,6,7)
\end{aligned}
$$

The product of sums expression will be given by POS form = Complement of $\Sigma(3,4)$

$$
\begin{aligned}
&= \text{Complement of } \Sigma\,(011,\ 100) \\
&= (\bar{A}BC + A\bar{B}\bar{C}) \\
&= \overline{(ABC + ABC)} \\
&= (\overline{\bar{A}BC} \cdot \overline{A\bar{B}\bar{C}}) \\
&= (\bar{\bar{A}} + \bar{B} + \bar{C})(\bar{A} + \bar{\bar{B}} + \bar{\bar{C}}) \\
&= (A + \bar{B} + \bar{C})(\bar{A} + B + C)
\end{aligned}
$$

When any switching expression is to be expressed in canonical POS form, then each term of the expression should be examined and if it is a maxterm, then it should be kept as it is. Thus if any term contains all the variables in either complemented or uncomplemented form, then it should be retained. If any particular variable does not occur in any sum term, then for each variable A, B or C which do not occur, add $A\bar{A}$, $B\bar{B}$ or $C\bar{C}$ as the case may be. Then convert the sum terms into product of sums and eliminate the repeated terms.

EXAMPLE 32: Final the canonical product of sums (POS) form and sum of products (SOP) form of the switching functions given below:

$$f(A,B,C) = (\bar{A}) \cdot (B + \bar{C})$$

Solution:

$$f(A,B,C) = (\bar{A}) \cdot (B + \bar{C})$$

In the first factor $(\bar{A})$, the variable B and C are absent and in the second factor $(B + \bar{C})$, the variable A is absent. Therefore adding $B\bar{B}$ and $C\bar{C}$ to the first factor and adding $A\bar{A}$ to the second factor, we get:

$$
\begin{aligned}
f(A,B,C) &= (\bar{A} + B\bar{B} + C\bar{C})(B + \bar{C} + A\bar{A}) \\
&= [(\bar{A} + B)(\bar{A} + \bar{B}) + C\bar{C}] \cdot [(B + \bar{C} + A)(B + \bar{C} + \bar{A})] \\
&= [\{(\bar{A} + B)(\bar{A} + \bar{B}) + C\}\{(\bar{A} + B)(\bar{A} + \bar{B}) + \bar{C}\}] \cdot [(A + B + \bar{C})(\bar{A} + B + \bar{C})] \\
&= [C + (\bar{A} + B) \cdot (\bar{A} + \bar{B})][\bar{C} + (\bar{A} + B) \cdot (\bar{A} + \bar{B})][(A + B + \bar{C})(\bar{A} + B + \bar{C})] \\
&= (C + \bar{A} + B) \cdot (C + \bar{A} + \bar{B}) \cdot (\bar{C} + \bar{A} + B) \cdot (\bar{C} + \bar{A} + \bar{B}) \cdot (A + B + \bar{C}) \cdot (\bar{A} + B + \bar{C}) \\
&= (\bar{A} + B + C)(\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})(A + B + \bar{C})(\bar{A} + B + \bar{C})
\end{aligned}
$$